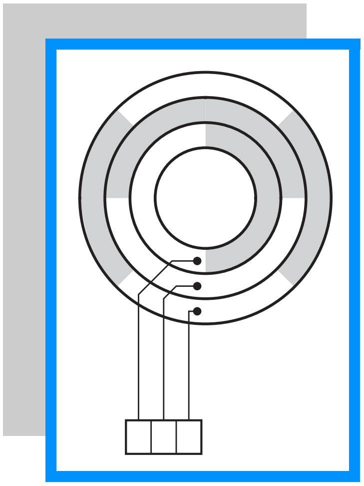

2

NUMBER SYSTEMS AND CODES

EXERCISE SOLUTIONS

2.1
(a) $1101011_2 = 6B_{16}$ (b) $174003_8 = 1111000000000011_2$
(c) $10110111_2 = B7_{16}$ (d) $67.24_8 = 110111.0101_2$
(e) $10100.1101_2 = 14.D_{16}$ (f) $F3A5_{16} = 1111001110100101_2$
(g) $11011001_2 = 331_8$ (h) $AB3D_{16} = 1010101100111101_2$
(i) $101111.0111_2 = 57.34_8$ (j) $15C.38_{16} = 101011100.00111_2$

2.3
(a) $1023_{16} = 1000000100011_2 = 10043_8$
(b) $7E6A_{16} = 111111001101010_2 = 77152_8$
(c) $ABCD_{16} = 1010101111001101_2 = 125715_8$
(d) $C350_{16} = 1100001101010000_2 = 141520_8$
(e) $9E36.7A_{16} = 1001111000110110.0111_2 = 117066.364_8$
(f) $DEAD.BEEF_{16} = 1101111010101101.1011111011101111_2 = 157255.575674_8$

2.5
(a) $1101011_2 = 107_{10}$ (b) $174003_8 = 63491_{10}$
(c) $10110111_2 = 183_{10}$ (d) $67.24_8 = 55.3125_{10}$
(e) $10100.1101_2 = 20.8125_{10}$ (f) $F3A5_{16} = 62373_{10}$
(g) $12010_3 = 138_{10}$ (h) $AB3D_{16} = 43837_{10}$
(i) $7156_8 = 3694_{10}$ (j) $15C.38_{16} = 348.21875_{10}$

2.6
(a) $125_{10} = 1111101_2$ (b) $3489_{10} = 6641_8$
(c) $209_{10} = 11010001_2$ (d) $9714_{10} = 22762_8$
(e) $132_{10} = 1000100_2$ (f) $23851_{10} = 5D2B_{16}$
(g) $727_{10} = 10402_5$ (h) $57190_{10} = DF66_{16}$
(i) $1435_{10} = 2633_8$ (j) $65113_{10} = FE59_{16}$

2.7
(a)
$$\begin{array}{r} 1100010 \\ 110101 \\ +\ 11001 \\ \hline 1001110 \end{array}$$
(b)
$$\begin{array}{r} 1011000 \\ 101110 \\ -\ 100101 \\ \hline 1010011 \end{array}$$
(c)
$$\begin{array}{r} 111111110 \\ 11011101 \\ +\ \ 1100011 \\ \hline 101000000 \end{array}$$
(d)
$$\begin{array}{r} 11000000 \\ 1110010 \\ +\ 1101101 \\ \hline 11011111 \end{array}$$

2.10
(a)
$$\begin{array}{r} 1372 \\ +\ 4631 \\ \hline 59A3 \end{array}$$
(b)
$$\begin{array}{r} 4F1A5 \\ +\ B8D5 \\ \hline 5AA7A \end{array}$$
(c)
$$\begin{array}{r} F35B \\ +\ 27E6 \\ \hline 11B41 \end{array}$$
(d)
$$\begin{array}{r} 1B90F \\ +\ C44E \\ \hline 27D5D \end{array}$$

2.11

decimal	+ 18	+ 115	+79	–49	–3	–100
signed-magnitude	00010010	01110011	01001111	10110001	10000011	11100100
two’s-magnitude	00010010	01110011	01001111	11001111	11111101	10011100
one’s-complement	00010010	01110011	01001111	11001110	11111100	10011011

2.18

$$h_j = \sum_{i=0}^{3} b_{4j+i} \cdot 2^j$$

Therefore,

$$B = \sum_{i-0}^{4n-1} b_i \cdot 2^i = \sum_{i=0}^{n-1} h_i \cdot 16^i$$

$$-B = 2^{4n} - \sum_{i=0}^{4n-1} b_i \cdot 2^i = 16^n - \sum_{i=0}^{n-1} h_i \cdot 16^i$$

Suppose a $3n$-bit number B is represented by an n-digit octal number Q. Then the two’s-complement of B is represented by the 8’s-complement of Q.

2.22 Starting with the arrow pointing at any number, adding a positive number causes overflow if the arrow is advanced through the +7 to –8 transition. Adding a negative number to any number causes overflow if the arrow is not advanced through the +7 to –8 transition.

2.24 Let the binary representation of X be $x_{n-1}x_{n-2}\ldots x_1x_0$. Then we can write the binary representation of Y as $x_mx_{m-1}\ldots x_1x_0$, where $m = n-d$. Note that x_{m-1} is the sign bit of Y. The value of Y is

$$Y = -2^{m-1} \cdot x_{m-1} + \sum_{i=0}^{n-2} x_i \cdot 2^i$$

The value of X is

$$\begin{aligned} X &= -2^{n-1} \cdot x_{n-1} + \sum_{i=0}^{n-2} x_i \cdot 2^i \\ &= -2^{n-1} \cdot x_{n-1} + Y + 2^{m-1} \cdot x_{m-1} + \sum_{i=m-1}^{n-2} x_i \cdot 2^i \\ &= -2^{n-1} \cdot x_{n-1} + Y + 2 \cdot 2^{m-1} + \sum_{i=m}^{n-2} x_i \cdot 2^i \end{aligned}$$

Case 1 $(x_{m-1} = 0)$ In this case, $X = Y$ if and only if $-2^{n-1} \cdot x_{n-1} + \sum_{i=m}^{n-2} x_i \cdot 2^i = 0$, which is true if and only if all of the discarded bits $(x_m \ldots x_{n-1})$ are 0, the same as x_{m-1}.

Case 2 $(x_{m-1} = 1)$ In this case, $X = Y$ if and only if $-2^{n-1} \cdot x_{n-1} + 2 \cdot 2^{m-1} + \sum_{i=m}^{n-2} x_i \cdot 2^i = 0$, which is true if and only if all of the discarded bits $(x_m \ldots x_{n-1})$ are 1, the same as x_{m-1}.

2.25 If the radix point is considered to be just to the right of the leftmost bit, then the largest number is $1.11 \cdots 1$ and the 2's complement of D is obtained by subtracting it from 2 (singular possessive). Regardless of the position of the radix point, the 1s' complement is obtained by subtracting D from the largest number, which has all 1s (plural).

2.28

$$B = -b_{n-1} \cdot 2^{n-1} + \sum_{i=0}^{n-2} b_i \cdot 2^i$$

$$2B = -b_{n-1} \cdot 2^{n} + \sum_{i=0}^{n-2} b_i \cdot 2^{i+1}$$

Case 1 $(b_{n-1} = 0)$ First term is 0, summation terms have shifted coefficients as specified. Overflow if $b_{n-2} = 1$.

Case 2 $(b_{n-1} = 1)$ Split first term into two halves; one half is cancelled by summation term $b_{n-2} \cdot 2^{n-1}$ if $b_{n-2} = 1$. Remaining half and remaining summation terms have shifted coefficients as specified. Overflow if $b_{n-2} = 0$.

2.32 001–010, 011–100, 101–110, 111–000.

2.34 Perhaps the designers were worried about what would happen if the aircraft changed altitude in the middle of a transmission. With the Gray code, the codings of "adjacent" alitudes (at 50-foot increments) differ in only one bit. An altitude change during transmission affects only one bit, and whether the changed bit or the original is transmitted, the resulting code represents an altitude within one step (50 feet) of the original. With a binary code, larger altitude errors could result, say if a plane changed from 12,800 feet (000100000000_2) to 12,750 feet (000011111111_2) in the middle of a transmission, possibly yielding a result of 25,500 feet (000111111111_2).

2–4 DIGITAL CIRCUITS

2.37