

Minimización de funciones de Conmutación

(Parte II)

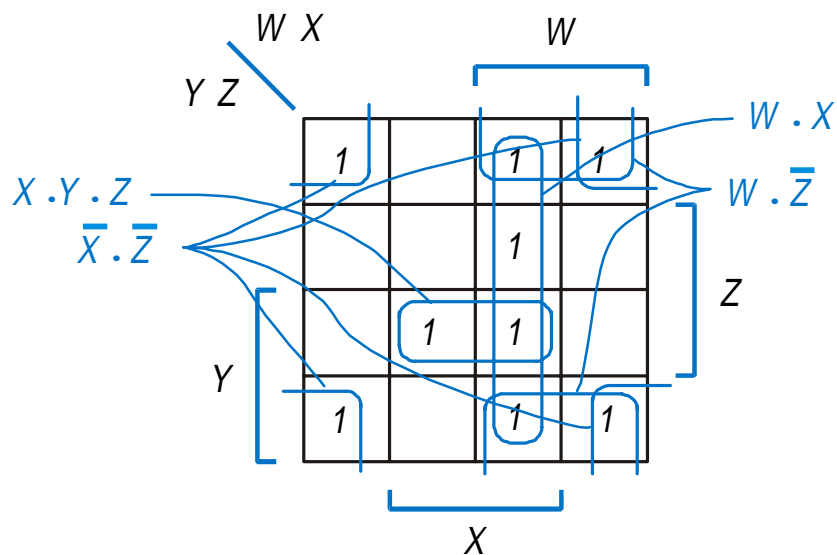
Prof. Luis Araujo

Sistemas Digitales

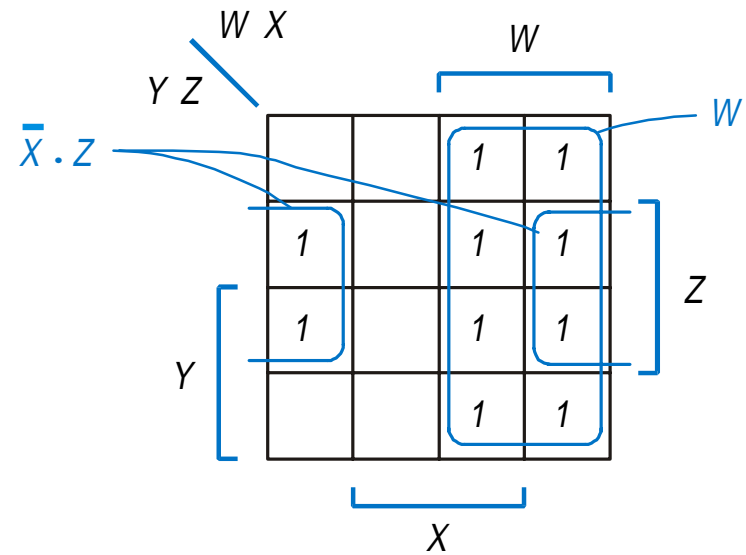
<http://www.ing.ula.ve/~araujol/sd>

Minimización por Mapas de Karnaugh

Suma Total: Suma de los implicants primos



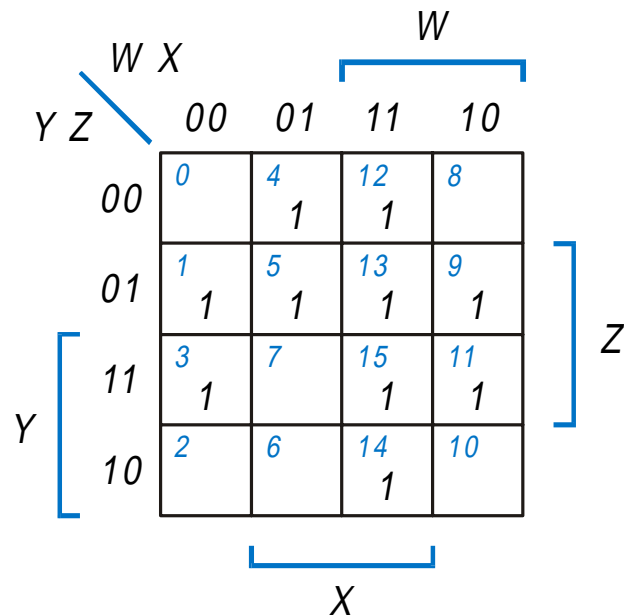
$$f(w, x, y, z) = x \cdot y \cdot z + \bar{x} \cdot \bar{z} + w \cdot x + w \cdot \bar{z}$$



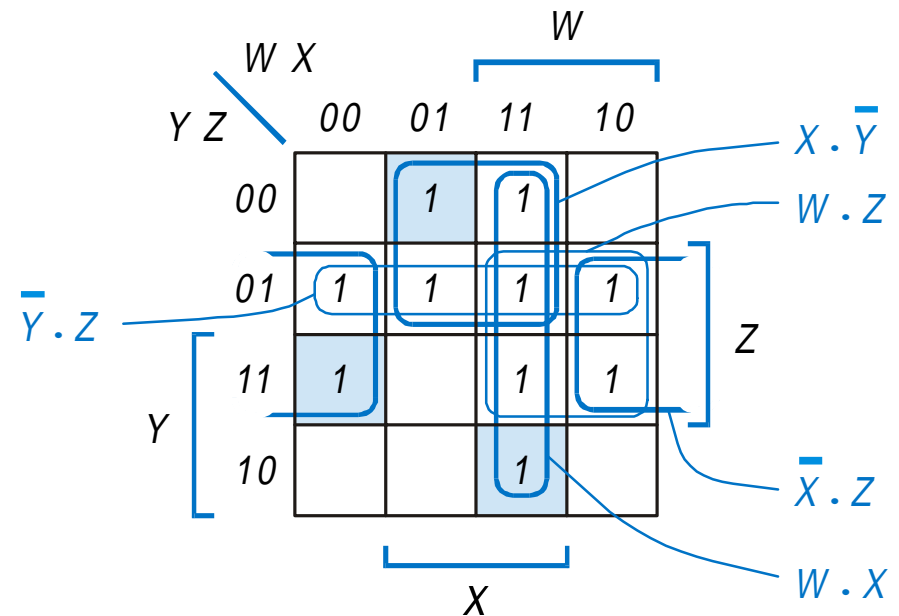
$$f(w, x, y, z) = w + \bar{x} \cdot z$$

Minimización por Mapas de Karnaugh

- Celdas 1 distinguidas: celdas 1 que están cubiertas por un único implicante primo.
- Implicante primo esencial(IPE): implicante que contenga al menos una celda 1 distinguida
- Suma Mínima: Suma de los IPE.



$$F(W,X,Y,Z) = \sum m(1,3,4,5,9,11,12,13,14,15)$$



$$F = X \cdot \bar{Y} + \bar{X} \cdot Z + W \cdot X$$

Minimización por Mapas de Karnaugh

W X		W			
		00	01	11	10
Y Z	00	0	4 1	12	8
	01	1	5 1	13 1	9
	11	3 1	7 1	15 1	11 1
	10	2 1	6 1	14	10

X

Z

$$F(W,X,Y,Z) = \sum m(2,3,4,5,6,7,11,13,15)$$

W X		W			
		00	01	11	10
Y Z	00		1		
	01		1	1	
	11	1	1	1	1
	10	1	1		

X

Z

$\bar{W}.Y$

$\bar{W}.X$

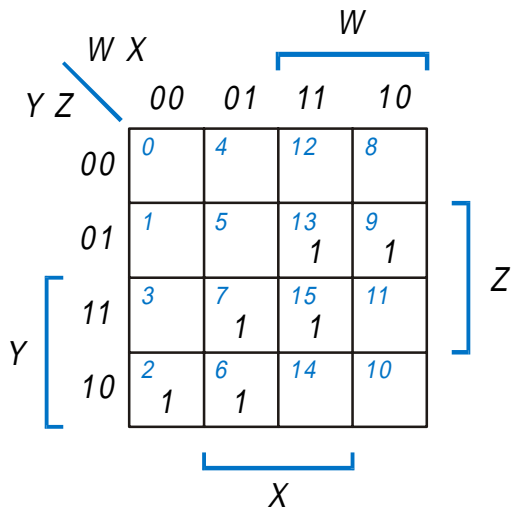
$X.Z$

$Y.Z$

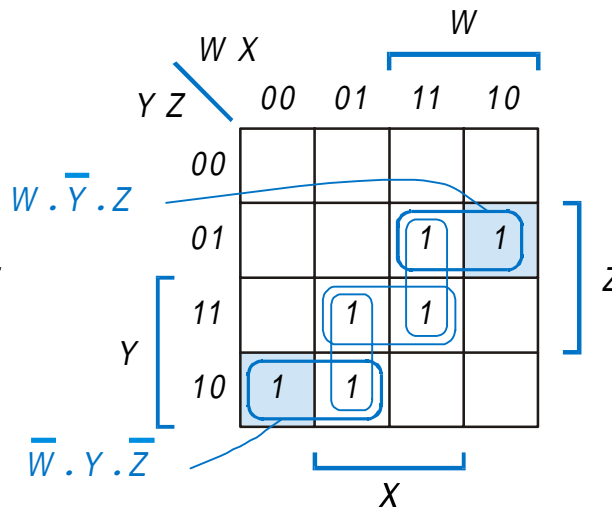
$$F = \bar{W}.Y + \bar{W}.X + X.Z + Y.Z$$

Minimización por Mapas de Karnaugh

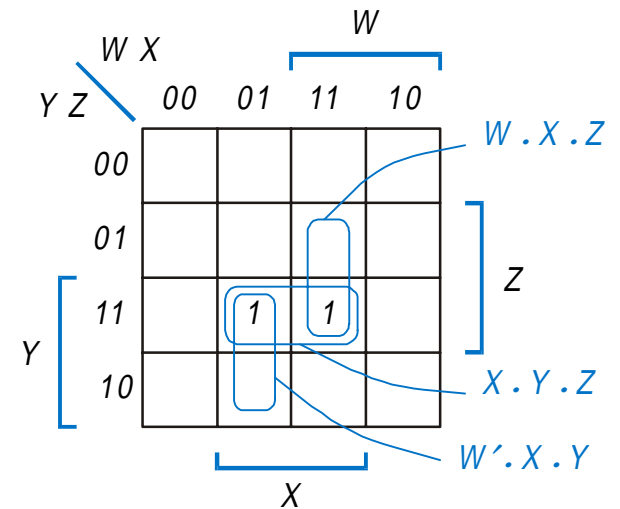
- Implicantes primos esenciales secundarios (IPES),
- Suma Mínima = IPE + IPES



$$F(W,X,Y,Z) = \Sigma m(2,6,7,9,13,15)$$



$$F = W \cdot \bar{Y} \cdot Z + \bar{W} \cdot Y \cdot \bar{Z} + X \cdot Y \cdot Z$$



Minimización por Mapas de Karnaugh

W X		W			
		00	01	11	10
Y Z	00	0 1	4 1	12 1	8 1
	01	1 1	5 1	13 1	9 1
Y	11	3 1	7 1	15 1	11 1
	10	2 1	6 1	14 1	10 1

X

$$F(W,X,Y,Z) = \Sigma m(0,1,2,3,4,5,7,14,15)$$

W X		W			
		00	01	11	10
Y Z	00	1	1		
	01	1	1		
Y	11	1	1	1	
	10	1		1	

X

$\bar{W} \cdot \bar{Y}$ (points to top-left 2x2 group)
 $\bar{W} \cdot \bar{X}$ (points to left column)
 $W \cdot X \cdot Y$ (points to center 1x1 group)

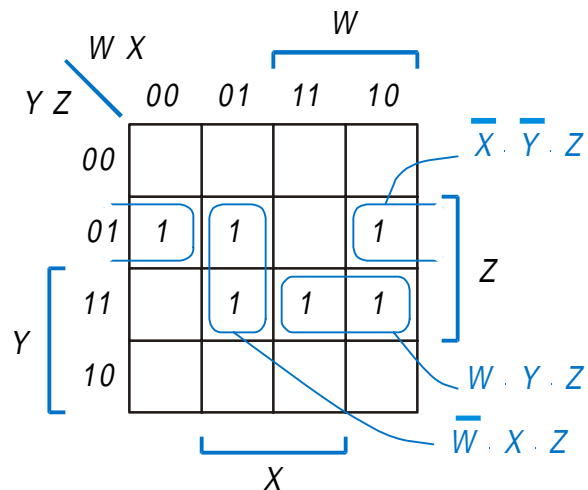
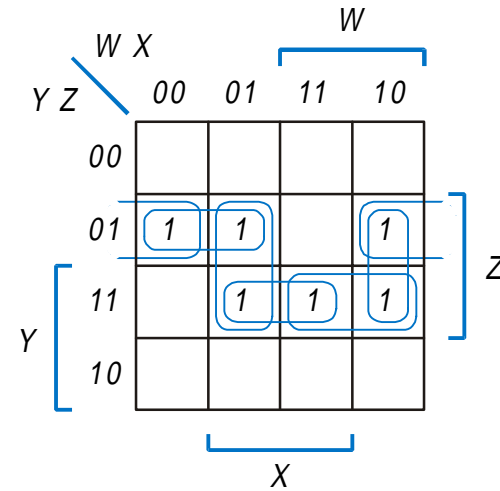
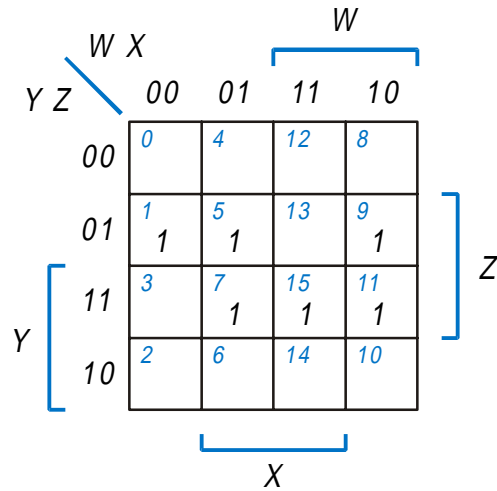
$$F = \bar{W} \cdot \bar{Y} + \bar{W} \cdot \bar{X} + W \cdot X \cdot Y + \bar{W} \cdot Z$$

W X		W			
		00	01	11	10
Y Z	00				
	01				
Y	11		1		
	10				

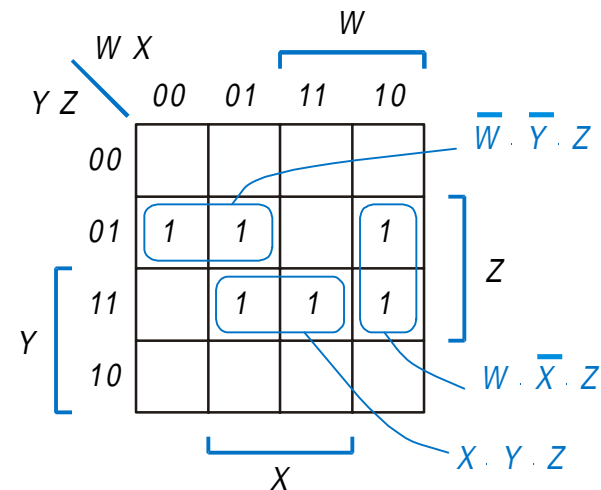
X

$\bar{W} \cdot Z$ (points to top row)
 $X \cdot Y \cdot Z$ (points to the single 1)

Minimización por Mapas de Karnaugh



$$F = \bar{W} \cdot X \cdot Z + W \cdot Y \cdot Z + \bar{X} \cdot \bar{Y} \cdot Z$$



$$F = X \cdot Y \cdot Z + W \cdot \bar{X} \cdot Z + \bar{W} \cdot \bar{Y} \cdot Z$$

Minimización por Mapas de Karnaugh

Condiciones No importa (“-” ó “d”)

W X		W			
		00	01	11	10
Y Z	00	0	4	12 d	8
	01	1 1	5 1	13 d	9
	11	3 1	7 1	15 d	11 d
	10	2 1	6	14 d	10 d

Diagram illustrating a 4x4 Karnaugh map for variables W, X, Y, and Z. The map shows minterms 0 through 15, with 'd' representing don't care conditions. The map is labeled with W, X, Y, and Z axes. Blue brackets indicate groupings for X and Z.

$$F(W,X,Y,Z) = \sum m(1,2,3,5,7) + d(10,11,12,13,14,15)$$

W X		W			
		00	01	11	10
Y Z	00			d	
	01	1	1	d	
	11	1	1	d	d
	10	1		d	d

Diagram illustrating the same 4x4 Karnaugh map with blue circles highlighting the prime implicants $\overline{W} \cdot Z$ and $\overline{X} \cdot Y$. The map is labeled with W, X, Y, and Z axes. Blue brackets indicate groupings for X and Z.

$$F = \overline{W} \cdot Z + \overline{X} \cdot Y$$

Minimización por Mapas de Karnaugh

W X		W			
		00	01	11	10
Y Z	00	0 0	4	12 0	8 0
	01	1 0	5	13	9 d
	11	3 d	7 d	15	11 0
	10	2 d	6 0	14 d	10 0

Diagram illustrating a 4x4 Karnaugh map for variables W, X, Y, and Z. The map shows the values of the function F(W,X,Y,Z) for each combination of W, X, Y, and Z. The map is labeled with W, X, Y, and Z axes. The values are: 0, 4, 12, 8, 1, 5, 13, 9, 3, 7, 15, 11, 2, 6, 14, 10. The map is also labeled with X and Z axes.

$$F(W,X,Y,Z) = \sum m(4,5,13,15) + d(2,3,7,9,14)$$

$$F(W,X,Y,Z) = \prod M(0,1,6,8,10,11,12) \cdot d(2,3,7,9,14)$$

W X		W			
		00	01	11	10
Y Z	00	0		0	0
	01	0			d
	11	d	d		0
	10	d	0	d	0

Diagram illustrating a 4x4 Karnaugh map for variables W, X, Y, and Z. The map shows the values of the function F(W,X,Y,Z) for each combination of W, X, Y, and Z. The map is labeled with W, X, Y, and Z axes. The values are: 0, 0, 0, 0, 0, d, d, 0, d, 0, d, 0. The map is also labeled with X and Z axes. The map is also labeled with W + Y-bar and Y-bar + Z.

$$F = X \cdot (\bar{W} + Z) \cdot (\bar{Y} + Z) \quad \text{ó} \quad X \cdot (\bar{W} + Z) \cdot (W + \bar{Y})$$

Minimización por Mapas de Karnaugh

- Para 5 variables:

W X		W			
		00	01	11	10
Y Z	00	0	4	12	8
	01	1	5	13	9
	11	3	7	15	11
	10	2	6	14	10

X

V=0

W X		W			
		00	01	11	10
Y Z	00	16	20	28	24
	01	17	21	29	25
	11	19	23	31	27
	10	18	22	30	26

X

V=1

Minimización por Mapas de Karnaugh

- Para 6 variables:

Diagrama de Karnaugh para 6 variables ($U, V = 0, 0$).

		W X		W		
		00	01	11	10	
Y	Z	00	01	11	10	Z
	00	0	4	12	8	
	01	1	5	13	9	
	11	3	7	15	11	
	10	2	6	14	10	
				X		

$U, V = 0, 0$

Diagrama de Karnaugh para 6 variables ($U, V = 0, 1$).

		W X		W		
		00	01	11	10	
Y	Z	00	01	11	10	Z
	00	16	20	28	24	
	01	17	21	29	25	
	11	19	23	31	27	
	10	18	22	30	26	
				X		

$U, V = 0, 1$

Diagrama de Karnaugh para 6 variables ($U, V = 1, 0$).

		W X		W		
		00	01	11	10	
Y	Z	00	01	11	10	Z
	00	32	36	44	40	
	01	33	37	45	41	
	11	35	39	47	43	
	10	34	38	46	42	
				X		

$U, V = 1, 0$

Diagrama de Karnaugh para 6 variables ($U, V = 1, 1$).

		W X		W		
		00	01	11	10	
Y	Z	00	01	11	10	Z
	00	48	52	60	56	
	01	49	53	61	57	
	11	51	55	63	59	
	10	50	54	62	58	
				X		

$U, V = 1, 1$